Answers:

0. -1
1. -1
2. $(-\infty,-2)\cup\left(-2,-\frac{6}{5}\right)\cup\left[\frac{2}{3},\infty\right)$ (must be in interval notation)
3. $-\frac{744}{1105}$
4. 146
5. 6
6. 307
7. 204
8. π
9. -3
10. 28
11. $\frac{1009}{1010}$
12. 17
13. 0.479 (must be a decimal)
14. 280

Solutions:

0. $A=\sum_{n=91}^{179}\sin\left(n^\circ\right)=\sum_{n=1}^{89}\sin\left(n^\circ\right)$ (since the values are positive in both quadrants I and II)

$B=\sum_{n=91}^{179}\cos\left(n^\circ\right)=\sum_{n=1}^{89}\cos\left(180^\circ-n^\circ\right)=-\sum_{n=1}^{89}\sin\left(n^\circ\right)$ (due to cosine subtraction formula)

Therefore, $A=-B$, meaning $\frac{A}{B}=-1$.

1. The quotient of $\frac{3x^3-17x^2+x+15}{2x^2+9x+3}$ is $\frac{3}{2}x-\frac{61}{4}$, and the remainder is non-zero, so $A=\frac{3}{2}$ and $B=-\frac{61}{4}$. The quotient of $\frac{6x^5+2x^3-13x^2+9x-8}{3x^3+4x^2-8x-4}$ is $2x^2-\frac{8}{3}x+\frac{86}{9}$, and the remainder is non-zero, so $C=2$, $D=-\frac{8}{3}$, and $E=\frac{86}{9}$. Therefore, $A+2B+C+D+3E$ $=\frac{3}{2}+2\left(-\frac{61}{4}\right)+2+\left(-\frac{8}{3}\right)+3\left(\frac{86}{9}\right)=-1$

2. $g(x)=\sqrt{\frac{3x^2+4x-4}{5x^2+16x+12}}=\sqrt{\frac{(3x-2)(x+2)}{(5x+6)(x+2)}}$, and performing sign analysis on the radicand shows that it is positive on the intervals $(-\infty,-2)$ (all four factors are negative); $\left(-2,-\frac{6}{5}\right)$ (two left factors are negative, two right factors are positive); and $\left(\frac{2}{3},\infty\right)$ (all four factors are positive). On the interval $\left(-\frac{6}{5},\frac{2}{3}\right)$, the radicand is negative since $(3x-2)$ is negative while the other three factors are positive. Further, $\frac{2}{3}$ should be included in the answer since the radicand equals 0, while -2 and $-\frac{6}{5}$ should not be included since the radicand is undefined. Therefore, the domain of the function is $(-\infty,-2)\cup\left(-2,-\frac{6}{5}\right)\cup\left[\frac{2}{3},\infty\right)$.

3. First, let's find the sine and cosine values of the three angles, and we know that α is in quadrant I, β is in quadrant IV, and γ is in quadrant III. Further, to find those values for β and γ, we must use the cosine addition formula:

$\sin\alpha=\frac{4}{5}$, $\cos\alpha=\frac{3}{5}$, $\sin\beta=-\frac{12}{13}$, $\cos\beta=\frac{5}{13}$, $\sin\gamma=-\frac{15}{17}$, and $\cos\gamma=-\frac{8}{17}$

Therefore,

$$\cos(\alpha+\beta+\gamma)=\cos(\alpha+\beta)\cos\gamma-\sin(\alpha+\beta)\sin\gamma=\cos\alpha\cos\beta\cos\gamma-\sin\alpha\sin\beta\cos\gamma$$
$$-\sin\alpha\cos\beta\sin\gamma-\cos\alpha\sin\beta\sin\gamma=\left(\frac{3}{5}\right)\left(\frac{5}{13}\right)\left(-\frac{8}{17}\right)-\left(\frac{4}{5}\right)\left(-\frac{12}{13}\right)\left(-\frac{8}{17}\right)$$
$$-\left(\frac{4}{5}\right)\left(\frac{5}{13}\right)\left(-\frac{15}{17}\right)-\left(\frac{3}{5}\right)\left(-\frac{12}{13}\right)\left(-\frac{15}{17}\right)=-\frac{120}{1105}-\frac{384}{1105}+\frac{300}{1105}-\frac{540}{1105}=-\frac{744}{1105}.$$

4. $$A=\sum_{i=7}^{18}\left(i^2\right)=\sum_{i=1}^{18}\left(i^2\right)-\sum_{i=1}^{6}\left(i^2\right)=\frac{18\cdot19\cdot37}{6}-\frac{6\cdot7\cdot13}{6}=2109-91=2018$$

$$B=\sum_{i=1}^{100}\left((2i-1)^3\right)=\sum_{i=1}^{200}\left(i^3\right)-\sum_{i=1}^{100}\left((2i)^3\right)=\frac{200^2\cdot201^2}{4}-8\cdot\frac{100^2\cdot101^2}{4}=404010000$$
$$-204020000=199990000$$

$\frac{199990000}{2018}=99103+\frac{146}{2018}$, so the remainder is 146.

5. $0=9^x-3^{x+2}+18=\left(3^x\right)^2-9\cdot3^x+18=\left(3^x-3\right)\left(3^x-6\right)\Rightarrow x=1$ or $x=\log_3 6$, so $A=\log_3 6$.

$x^4+4x^3+4x^2-4x-5=(x-1)(x+1)(x+2+i)(x+2-i)$, so the greatest absolute value of any of these roots is $|-2+i|=\sqrt{(-2)^2+1^2}=\sqrt{5}$, meaning $B=\sqrt{5}$.

$$C=\lim_{h\to0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to0}\frac{3(1+h)^2-5(1+h)+1-(-1)}{h}=\lim_{h\to0}\frac{\cancel{h}(1+3h)}{\cancel{h}}=1$$

$$\left(B\cdot\frac{(C+B)^2+(C-B)^2}{(C+B)^2-(C-B)^2}\right)^A=\left(\sqrt{5}\cdot\frac{\left(1+\sqrt{5}\right)^2+\left(1-\sqrt{5}\right)^2}{\left(1+\sqrt{5}\right)^2-\left(1-\sqrt{5}\right)^2}\right)^{\log_3 6}=\left(\sqrt{5}\cdot\frac{12}{4\sqrt{5}}\right)^{\log_3 6}=3^{\log_3 6}=6$$

6. $\frac{5\cdot106+4\cdot32+3\cdot22+2\cdot5+1\cdot8}{173}=4.289...$, so $A=4.29$. There are more than 87 5s on the BC test, so $B=5$, and 5 was the most common score, so $C=5$.

$\frac{5\cdot104+4\cdot125+3\cdot120+2\cdot62+1\cdot71}{482}=3.267...$, so $D=3.27$. When the scores on the AB test are listed in increasing order, the 241st and 242nd numbers are both 3s, so $E=3$, and 4 was the most common score, so $F=4$.

$100(A+B+C+D+E+F)=100(24.56)=2456$, the prime factorization of which is $2^3\cdot 307$, so the largest prime factor is 307.

7. The males on the committee could be formed in one of three categories: (M1) two students; (M2) one student, one faculty member; or (M3) two faculty members. The females could be formed in one of three categories: (F1) three students, one faculty member; (F2) two students, two faculty members; or (F3) one student, three faculty members. The only categories and male and female members that adhere to the given criteria are the following seven: (M1F2), (M1F3), (M2F1), (M2F2), (M2F3), (M3F1), and (M3F2). Using combinatorics methods, here are the number of ways of forming each category:

(M1F2): $\binom{2}{2}\binom{3}{2}\binom{3}{2}=1\cdot 3\cdot 3=9$ (M1F3): $\binom{2}{2}\binom{3}{1}\binom{3}{3}=1\cdot 3\cdot 1=3$

(M2F1): $\binom{2}{1}\binom{4}{1}\binom{3}{3}\binom{3}{1}=2\cdot 4\cdot 1\cdot 3=24$ (M2F2): $\binom{2}{1}\binom{4}{1}\binom{3}{2}\binom{3}{2}=2\cdot 4\cdot 3\cdot 3=72$

(M2F3): $\binom{2}{1}\binom{4}{1}\binom{3}{1}\binom{3}{3}=2\cdot 4\cdot 3\cdot 1=24$ (M3F1): $\binom{4}{2}\binom{3}{3}\binom{3}{1}=6\cdot 1\cdot 3=18$

(M3F2): $\binom{4}{2}\binom{3}{2}\binom{3}{2}=6\cdot 3\cdot 3=54$

Therefore, the total number of committees is $9+3+24+72+24+18+54=204$.

8. Multiply both sides of the equation by *r* to get $r^2+163=16r\cos\theta+20r\sin\theta\Rightarrow x^2+y^2+163=16x+20y\Rightarrow(x-8)^2+(y-10)^2=1$, so the graph is a circle with radius 1, thus enclosing an area of π.

9. Keep in mind that $a^2+b^2=c^2$. $(Da-2b+2c)^2+(2a+Eb+2c)^2=(D^2a^2-4Dab+4Dac$ $+4b^2-8bc+4c^2)+(4a^2+4Eab+8ac+E^2b^2+4Ebc+4c^2)=(D^2+4)a^2+(4E-4D)ab$ $+(4D+8)ac+(E^2+4)b^2+(4E-8)bc+8c^2$. Further, $(2a-2b+Fc)^2=4a^2-8ab+4Fac$ $+4b^2-4Fbc+F^2c^2$. If you compare the corresponding parts, be careful not to compare the square terms as they seem to suggest two values of 0 and one irrational number (both of which violate the terms of the problem)—remember that square terms can be combined here. Instead, compare the terms with two distinct letters among *a*, *b*, and *c*:

$4E-4D=-8$, $4D+8=4F$, and $4E-8=-4F$. Solving this system yields $D=1$, $E=-1$, and $F=3$. Therefore, the product of *D*, *E*, and *F* is -3.

As an aside, this makes the Pythagorean triple $(a-2b+2c, 2a-b+2c, 2a-2b+3c)$. $(a+2b+2c, 2a+b+2c, 2a+2b+3c)$ and $(-a+2b+2c, -2a+b+2c, -2a+2b+3c)$ are also Pythagorean triples, given that (a,b,c) is a Pythagorean triple.

10. $A^2(B+C)+B^2(A+C)+C^2(A+B)=A^2(A+B+C)+B^2(A+B+C)+C^2(A+B+C)$ $-(A^3+B^3+C^3)=(A^2+B^2+C^2)(A+B+C)-(A^3+B^3+C^3)$. Therefore, we can use the method of Newton's sums to find these three values:

$$S_1=A+B+C=-5$$
$$0=S_2+5S_1-11\cdot 2=S_2+5(-5)-11\cdot 2\Rightarrow S_2=A^2+B^2+C^2=47$$
$$0=S_3+5S_2-11S_1-9\cdot 3=S_3+5\cdot 47-11(-5)-9\cdot 3\Rightarrow S_3=A^3+B^3+C^3=-263$$

Therefore, $S_2\cdot S_1-S_3=47(-5)-(-263)=28$.

11. The given equation can be rewritten as $(n-1)f(n)-1=0$, and because *f* is a polynomial of degree 2018, the left side of this new equation is a polynomial of degree 2019 whose roots are the integers from 2 to 2020, inclusive. Therefore, $(n-1)f(n)-1=a(n-2)(n-3)...(n-2020)$ for some real number *a*. Plugging $n=1$ into this equation yields $-1=a(-1)(-2)...(-2019)\Rightarrow a=\frac{1}{2019!}$, which now makes the equation $(n-1)f(n)-1=\frac{1}{2019!}(n-2)(n-3)...(n-2020)$. Plugging $n=2021$ into this equation yields $2020f(2021)-1=\frac{1}{2019!}(2019)(2018)...(1)=1\Rightarrow 2020f(2021)=2$ $\Rightarrow f(2021)=\frac{2}{2020}=\frac{1}{1010}$. Further, plugging $n=2022$ into this equation yields $2021f(2022)-1=\frac{1}{2019!}(2020)(2019)...(2)=2020\Rightarrow 2021f(2022)=2021$ $\Rightarrow f(2022)=1$. Therefore, $f(2022)-f(2021)=1-\frac{1}{1010}=\frac{1009}{1010}$.

12. We only need find the last two digits of each term in the series, since the remainder when divided by 100 equals the final two digits of the sum. For each term, the last two digits are the following (modular arithmetic may be used to find these values):

$1^1 : 01$	$2^2 : 04$	$3^3 : 27$	$4^4 : 56$	$5^5 : 25$
$6^6 : 56$	$7^7 : 43$	$8^8 : 16$	$9^9 : 89$	$10^{10} : 00$

$1+4+27+56+25+56+43+16+89=317$, so the last two digits of the sum are 17.

13. Let m and f be the number of left-handed male and female students, respectively, in the sixth grade at Mu Alpha Theta Middle School. This means that $9m$ and $7f$ are the number of right-handed male and female students, respectively. Therefore, $m+f=24$ and $9m+7f=190$. Solving this system yields $m=11$ and $f=13$. The probability that a right-handed student is female is $\frac{91}{190}=0.4789...$, so this probability, when rounded to the nearest thousandth, is 0.479.

14. A is the number of derangements of 4 items, so $A=9$ (the number of permutations of 4 items is small enough that these can be listed fairly easily). B is four times the number of derangements of 3 items (keeping each of four cars parked in its same-numbered garage, we must find the number of derangements of the other three cars), so $B=4\cdot 2=8$. Therefore, $A\cdot B+A+B+19=9\cdot 8+9+8+19=108=3^3\cdot 2^2$, so the number of positive integral divisors of 108 is $\left(1+3+3^2+3^3\right)\left(1+2+2^2\right)=40\cdot 7=280$.

Cars in different garages	One car in same garage	Divisors of 108
2143	1342	1
2341	1423	2
2413	2314	3
3142	2431	4
3412	3124	6
3421	3241	9
4123	4132	12
4312	4213	18
4321		27
		36
		54
		+108
		280